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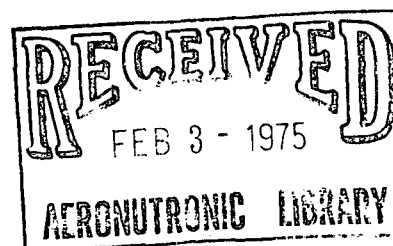
**DESIGN OF EXPERIMENTS FOR MEASURING  
HEAT-TRANSFER COEFFICIENTS WITH  
A LUMPED-PARAMETER CALORIMETER**

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16. Abstract A theoretical investigation was conducted to determine optimum experimental conditions for using a lumped-parameter calorimeter to measure heat-transfer coefficients and heating rates. A mathematical model of the transient temperature response of the calorimeter was used with the measured temperature response to predict the heat-transfer coefficient and the rate of heating. A sensitivity analysis was used to determine the optimum transient experiment for simultaneously measuring the heat addition during heating and the convective heat-transfer coefficient during heating and cooling of a lumped-parameter calorimeter. Optimum experiments were also designed for measuring the convective heat-transfer coefficient during both heating and cooling and cooling only.			
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# DESIGN OF EXPERIMENTS FOR MEASURING HEAT-TRANSFER COEFFICIENTS WITH A LUMPED-PARAMETER CALORIMETER

by G. James Van Fossen, Jr.

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## SUMMARY

A theoretical investigation was conducted to determine optimum experimental conditions for using a lumped-parameter calorimeter to measure heat-transfer coefficients and heating rates. A mathematical model of the transient temperature response of the calorimeter was used with the measured temperature response to predict the heat-transfer coefficient and the rate of heating.

A sensitivity analysis was used to find optimum experimental conditions for the lumped-parameter calorimeter. An optimum transient experiment was designed for simultaneously measuring the heat addition during heating and the convective heat-transfer coefficient during heating and cooling of the calorimeter for steady convective conditions.

Optimum experiments were also designed for measuring the convective heat-transfer coefficient during both heating and cooling and cooling only.

For the case of measuring the heat-transfer coefficient from the cooling of the calorimeter only, the optimum experiment consists of allowing the dimensionless temperature of the calorimeter to fall from 1.0 to about 0.183. For this case the rate of heat addition to the calorimeter during heating has no effect on the optimum experiment.

For the other two cases, that is, the cases where both heating and cooling data of the calorimeter were used, it was shown that the optimum experiment was dependent on dimensionless heating rate. A heating rate as slow as practical was shown to be the best. The optimum dimensionless duration of the experiment was shown to be dependent on heating rate for the case of measuring both heat-transfer coefficient and heating rate simultaneously and the case of measuring heat-transfer coefficient only.

## INTRODUCTION

An analytical investigation based on sensitivity coefficients was conducted to determine the optimum transient experiment for measuring convective heat-transfer coefficients with a lumped-parameter calorimeter (LPC). Lumped-parameter calorimeters consist of a high-thermal-conductivity material having a known specific heat. They can be imbedded in the surface of a larger body to measure local heat-transfer coefficients. The usual approach for these transient tests is to have the gas stream conditions at steady state and to heat the calorimeter electrically to a predetermined temperature above that of the surrounding surface. The heater is then switched off, and the calorimeter is allowed to cool by convection. By measuring the temperature of the calorimeter as a function of time during the heating and cooling cycles, it is possible to determine, by use of a mathematical model, the heat added by the electric heater and the heat-transfer coefficient on the surface at the location of the LPC.

Heat-transfer investigations that have used the LPC approach are reported in references 1 to 3. In all three references, only the cooling portions of the cycle were used for determining convective heat-transfer coefficients. In addition, the coefficients in references 1 and 2 were based upon only two temperature measurements - the initial temperature of the LPC prior to beginning the cooling portion of the cycle, and a single time-temperature measurement during the cooling cycle. The coefficients obtained in reference 3 were based upon the slope of the experimentally measured time-temperature curve.

The present investigation was conducted to show the optimum time for making temperature measurements and to introduce the method of least squares for utilizing multiple time-temperature measurements during both the heating and cooling cycles for increased accuracy in heat-transfer-coefficient and heat-addition measurements. Optimum experiments are defined for the following three cases: (1) heat-transfer coefficients calculated by using cooling data only, (2) heat-transfer coefficients calculated by using both heating and cooling data, and (3) both heat-transfer coefficients and heat addition to the calorimeter during the heating cycle calculated simultaneously from heating and cooling data.

## THEORY

In this section a mathematical model is developed for the LPC. It is also shown that the least-square method can be used to calculate the convective heat-transfer coefficient from the data and the mathematical model.

The LPC is usually made of a high-thermal-conductivity material such as copper or aluminum. The high thermal conductivity helps to promote a uniform temperature

distribution. The assumption of uniform temperature allows the calorimeter to be lumped into a single mass. This simplifies the mathematical modeling of the calorimeter.

In order to maintain a uniform temperature in the calorimeter, the Biot number must be kept less than about 0.1 (ref. 4). The Biot number is given by

$$Bi = \frac{hL}{k} \quad (1)$$

(All symbols are defined in the appendix.)

Equation (1) shows the importance of using a high-thermal-conductivity material for the calorimeter. The characteristic length  $L$  is obtained by dividing the volume of the calorimeter by the convection surface area. Thus, the Biot number can also be controlled by the physical dimensions of the calorimeter.

The LPC has some means to input heat, usually an electric heater. A thermocouple is usually used to measure the temperature of the calorimeter. The operation of the LPC is quite simple. The convective flow is established, the heater is turned on, and the calorimeter is heated to some temperature above ambient. When the predetermined maximum temperature is reached, the heater is switched off and the LPC is allowed to cool by convection. The temperature is recorded as a function of time during both heating and cooling. A mathematical model is then used to estimate the convective heat-transfer coefficient.

### Mathematical Model

The assumptions used for a mathematical model of the LPC are

- (1) Temperature in the calorimeter material is uniform.
- (2) Heat-transfer coefficient remains constant throughout both heating and cooling of the LPC.
- (3) Calorimeter material properties are known.
- (4) Ambient temperature remains constant.
- (5) Air properties remain constant.
- (6) Heat addition to calorimeter is constant until heater is switched off.

With these assumptions a simple heat balance gives the dimensionless temperature history of the LPC as

$$\Phi = \begin{cases} \beta(1 - e^{-\tau}) & \tau \leq \tau_0 \\ e^{-(\tau - \tau_0)} & \tau \geq \tau_0 \end{cases} \quad (2)$$

$$\Phi = \begin{cases} \beta(1 - e^{-\tau}) & \tau \leq \tau_0 \\ e^{-(\tau - \tau_0)} & \tau \geq \tau_0 \end{cases} \quad (3)$$

where  $\tau_0$  is the dimensionless time at which the heater is switched off. From equations (2) and (3)  $\tau_0$  must be

$$\tau_0 = -\ln \left( 1 - \frac{1}{\beta} \right) \quad (4)$$

The dimensionless temperature  $\Phi$  is defined as

$$\Phi = \frac{T - T_s}{\Delta T_{\max}} \quad (5)$$

where  $\Delta T_{\max}$  is the maximum temperature rise above ambient. This temperature rise is determined before the experiment starts and is an experimental constraint; the heater is not switched off until  $\Delta T_{\max}$  is attained.

The dimensionless heating rate  $\beta$  is defined as

$$\beta = \frac{Q}{hA \Delta T_{\max}} \quad (6)$$

Note that if  $\beta$  is less than or equal to 1.0,  $\Delta T_{\max}$  will never be attained. The dimensionless time  $\tau$  is given by

$$\tau = \frac{ht}{\rho LC_p} \quad (7)$$

Figure 1 shows dimensionless temperature as a function of dimensionless time for several dimensionless heating rates.

### Calculation of Heat-Transfer Coefficient

One method that is used to calculate the heat-transfer coefficient from the data is to use a single time-temperature measurement during the cooling of the LPC. The

temperature of the LPC above ambient is assumed to be known at time  $\tau_0$ . The experimentally measured temperature and time at some time greater than  $\tau_0$  are inserted into equation (3), and the heat-transfer coefficient is determined from the resulting equation.

A more accurate method of calculating  $h$  from the LPC would be to use more than one data point. The method of least squares allows the use of any number of data points in the calculation. The method of least squares consists of minimizing the least-square function

$$F(h) = \sum_{i=1}^N [T_i - T_{i,cal}(h)]^2 \quad (8)$$

with respect to the unknown heat-transfer coefficient  $h$ . The value of  $h$  that minimizes  $F(h)$  is the best value.

The method of least squares allows all the data to be used. With this method, data from the heating portion of the cycle as well as the cooling data can be used. If both the heating and cooling data are used, it is also possible to simultaneously calculate the heating rate and the heat-transfer coefficient.

## ANALYSIS OF OPTIMUM EXPERIMENT

An optimum experiment is defined as one which allows the parameters in a mathematical model to be calculated from data with the greatest accuracy. One might think that experimentalists always perform experiments that will give the greatest accuracy; however, this is not true in the sense meant here. One can run the best experiment in terms of careful specimen preparation, placement of sensors, test procedure, and data acquisition and still have a poor experiment for measuring parameters. It is demonstrated by example later in this section that errors in the calculated heat-transfer coefficients caused by errors in the data can be amplified if measurements are taken at the wrong time. The effect of errors in the data can be minimized if measurements are taken at the optimum time.

The experimental conditions which yield predicted temperatures that are most sensitive to changes in the parameters are the optimum conditions. The experimental conditions which can be changed (controllable variables) in LPC tests are maximum temperature rise, characteristic length of the calorimeter, heating rate, duration of the experiment, and calorimeter material.

Sensitivity coefficients can be used to find the optimum experimental conditions. A sensitivity coefficient is defined as the derivative of the predicted value of a measured

quantity with respect to the parameter to be estimated. Sensitivity coefficients are made dimensionless by multiplying by the parameter and dividing by a function of the measured quantity. Sensitivity coefficients show how the quantity predicted from the mathematical model is affected by small changes in the parameters.

For the LPC the parameters we wish to estimate are the convective heat-transfer coefficient  $h$  and the heating rate  $Q$ . For the case of predicting  $h$  from the cooling of the calorimeter, the sensitivity coefficient is obtained from equation (3) by differentiating with respect to  $h$ . The time  $\tau_0$  is set to zero because the heating cycle does not affect the temperature for this case. We obtain in dimensionless form

$$S_h^T = \frac{h}{\Delta T_{\max}} \frac{\partial T}{\partial h} = -\tau e^{-\tau} \quad (9)$$

Figure 2 shows the dimensionless sensitivity coefficient for this case as a function of dimensionless time.

For both heating and cooling the dimensionless sensitivity coefficient  $S_h^T$  is given by

$$S_h^T = \begin{cases} -\beta(1 - e^{-\tau}) + \beta\tau e^{-\tau} & \tau \leq \tau_0 \\ \left(\frac{1}{\beta - 1} - \tau\right) e^{-(\tau - \tau_0)} & \tau > \tau_0 \end{cases} \quad (10)$$

$$(11)$$

Figure 3(a) shows  $S_h^T$  as a function of  $\tau$ .

For both heating and cooling the dimensionless sensitivity coefficient  $S_Q^T$  is given by

$$S_Q^T = \frac{Q}{\Delta T_{\max}} \frac{\partial T}{\partial Q} = \begin{cases} \beta(1 - e^{-\tau}) & \tau \leq \tau_0 \\ -\left(\frac{1}{\beta - 1}\right) e^{-(\tau - \tau_0)} & \tau > \tau_0 \end{cases} \quad (12)$$

$$(13)$$

Figure 3(b) shows  $S_Q^T$  as a function of  $\tau$ .

Extending the work of Box and Lucas (ref. 5), Beck (ref. 6) proposed a criterion to find the optimum experimental conditions. For a fixed temperature range and a large number of equally spaced measurements in time, Beck's criterion is to maximize the determinant of the matrix  $\underline{D}$ . The elements of  $\underline{D}$  are given by

$$d_{ij} = \frac{1}{N\tau_m} \int_0^{\tau_m} (S_{p_i}^T)(S_{p_j}^T) d\tau \quad (14)$$



where  $\tau_m$  is the duration of the experiment. The maximum temperature rise is a constraint and must be reached sometime between zero and  $\tau_m$ .

The optimization procedure then is to fix all the controllable variables except  $\tau_m$  and calculate

$$\bar{D} = \det \underline{D} \quad (15)$$

as a function of  $\tau_m$ , where  $\bar{D}$  is the sensitivity criterion. The maximum value of  $\bar{D}$

$$\max_{\tau_m} [\bar{D}] = \bar{D}_{\text{opt}} \quad (16)$$

indicates the best value of  $\tau_m$  for that particular setting of the other controllable variables. This procedure is repeated for different settings of the other controllable variables until the best combination is found.

As an example of how the sensitivity criterion  $\bar{D}$  is formed, consider the case of predicting the two parameters  $Q$  and  $h$ . Let

$$p_1 = h \quad (17)$$

and

$$p_2 = Q \quad (18)$$

Then from equation (14) the elements of the matrix  $\underline{D}$  are

$$d_{11} = \frac{1}{N\tau_m} \int_0^{\tau_m} (S_h^T)^2 dt \quad (19)$$

$$d_{12} = d_{21} = \frac{1}{N\tau_m} \int_0^{\tau_m} (S_h^T) (S_Q^T) dt \quad (20)$$

and

$$d_{22} = \frac{1}{N\tau_m} \int_0^{\tau_m} (S_Q^T)^2 dt \quad (21)$$

The sensitivity criterion  $\bar{D}$  is then

$$\bar{D} = d_{11}d_{22} - (d_{12})^2 \quad (22)$$

In order to demonstrate quantitatively the advantage of the least-square method over the single-point method, a sample case was constructed. The case of measuring the heat-transfer coefficient from cooling was chosen. The mathematical model for the dimensionless temperature is

$$\Phi = e^{-\alpha t} \quad (23)$$

The exact value of the parameter  $\alpha$  was taken as 1.0; thus, dimensionless time corresponded to real time. The effect of errors in the measured value of  $\Phi$  on the calculated value of the parameter  $\alpha$  was investigated by using the single-point method to calculate  $\alpha$  from simulated data. The simulated data were generated from the model (eq. (23)) by randomly adding or subtracting an error of 1 percent of the maximum value of  $\Phi$  (0.01) to the calculated  $\Phi$  for each time. The parameter  $\alpha$  was calculated from

$$\alpha = - \frac{\ln(\Phi \pm \text{error})}{t} \quad (24)$$

for each simulated data point.

The least-square method was also used to calculate  $\alpha$  from measurements containing errors. For this case the same 1 percent of maximum  $\Phi$  error was randomly added to or subtracted from the exact value of  $\Phi$  for seven data points that were equally spaced from 0.25 to 1.75 units of time.

## RESULTS

For the case of calculating  $h$  from cooling only, equation (3) shows that the only controllable variable which appears in the model is dimensionless time. For this case, only a single curve is necessary for the optimization. Figure 4 shows the sensitivity criterion as a function of dimensionless duration of experiment. The optimum dimensionless duration is about 1.7 after the heater is switched off. From the mathematical model (eq. (3)) this corresponds to letting the dimensionless temperature fall from 1.0 to about 0.183. For this case the sensitivity criterion is not affected by either the heating rate or the maximum temperature rise. The experimenter should let common sense dictate the maximum temperature rise. Too small a temperature rise will allow noise and other measurement errors to degrade the data.

For the case of calculating  $h$  from both the heating and cooling data the sensitivity criterion becomes a function of the dimensionless heating rate as well as the dimensionless duration of the experiment. Figure 5 shows the sensitivity criterion as a function of dimensionless duration of experiment for several values of dimensionless heating rate. From this figure it appears that having the dimensionless heating rate near unity is a desirable condition.

If the dimensionless heating rate  $\beta$  were unity, the LPC would take an infinite time to reach  $\Delta T_{\max}$ . The optimum dimensionless time  $\tau_{\text{opt}}$  as a function of  $\beta$  is shown in figure 6. The experimenter should choose  $\beta$  as near unity as possible, but he must maintain a reasonable real time  $t$ . The dimensionless time is given by equation (7).

The real-time duration of the experiment can be shortened for a given  $h$  by making  $L$  small. This is also consistent with the requirement of uniform temperature (eq. (1)).

The sensitivity criterion is again a function of both heating rate and dimensionless time for the case of calculating both  $Q$  and  $h$  from heating and cooling. Figure 7 shows the sensitivity criterion as a function of dimensionless time for several heating rates. The best experiment is again one in which  $\beta$  is near unity.

Figure 8 shows the optimum dimensionless time as a function of dimensionless heating rate for the case of calculating both  $h$  and  $Q$  from the heating and cooling data. The results are similar to the case of calculating  $h$  from heating and cooling data.

The results of the sample case used to demonstrate the advantage of the least-square method over the single-point method are shown in figure 9. The upper and lower curves in the figure show the errors in the parameter  $\alpha$  calculated by using the single-point method (eq. (24)) as a function of the time the measurement was taken. For the upper curve the error was subtracted from  $\Phi$ , and for the lower curve the error was added.

The data used to calculate the parameter  $\alpha$  with least squares are shown in figure 10. The circular symbols are the simulated data points, and the curve is the mathematical model of equation (23). The resulting error in  $\alpha$  calculated by least squares is shown in figure 9 as a constant over the range of measurement. The error is 0.226 percent. Figure 9 verifies that greater accuracy can be obtained with the least-square method than with the single-point method. Figure 9 also shows that with the single-point method the effect of measurement error can be minimized by choosing the correct time to take the measurement. The figure indicates that the error in the calculated value of  $\alpha$  is minimum when the time is 1.0. This is the same time at which the sensitivity coefficient shown in figure 2 is a maximum.

## CONCLUSIONS

A sensitivity analysis has been used to design the optimum experiment for a lumped-parameter calorimeter (LPC) used to measure heat-transfer coefficients. Three cases

have been considered. In the first case the LPC was used to measure the heat-transfer coefficient from cooling data only. Both the heating and cooling cycles were used to measure the heat-transfer coefficient in the second case. For this case the heat input during heating was assumed to be known. Finally, for the third case, both the heating and cooling data were used to measure both the heat-transfer coefficient and the heat input during heating.

The optimum conditions for the first case were not affected by the heating rate. The optimum dimensionless time for this case was about 1.7. This corresponds to letting the LPC cool off until the dimensionless temperature is about 0.183.

For the second case, both the heating and cooling cycles were used to measure the heat-transfer coefficient. The sensitivity criterion is affected by the heating rate for this case. The dimensionless heating rate should be chosen near unity. It may not be practical to choose the dimensionless heating rate too near unity because the real-time duration of the experiment may become too large. The real-time duration of the experiment can be controlled by varying the physical dimensions of the calorimeter. Once the heating rate is chosen, the optimum dimensionless duration of the experiment can be obtained. The mathematical model can then be used to translate this time into a temperature.

For the third case, both the heating and cooling cycles were used to measure the constant heat-transfer coefficient and the constant heating rate during heating. The optimum dimensionless heating rate was found to be near unity, as in the second case. The real-time duration of the experiment can be made reasonable by designing the calorimeter such that its characteristic length is small. Once the dimensionless heating rate is chosen, the optimum dimensionless time can be found.

A comparison of the single-point and multiple-point (least square) methods of measuring the heat-transfer coefficient from the cooling of the LPC shows that the least-square method results in less error. The error in the heat-transfer coefficient calculated by using the single-point method can be minimized if the measurement is taken at the time when the sensitivity coefficient is maximum, that is, when dimensionless time is 1.0.

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National Aeronautics and Space Administration,  
and  
U.S. Army Air Mobility R&D Laboratory,  
Cleveland, Ohio, November 5, 1974,  
505-04.

## APPENDIX - SYMBOLS

$A$	area for convection
$Bi$	Biot number, $hL/k$
$C_p$	specific heat of calorimeter material
$\underline{D}$	sensitivity matrix whose elements are $d_{ij}$
$\bar{D}$	determinant of matrix $\underline{D}$
$\bar{D}_{opt}$	maximum value of $\bar{D}$ as defined by eq. (16)
$d_{ij}$	elements of $\underline{D}$ defined by eq. (12)
$h$	convective heat-transfer coefficient
$k$	thermal conductivity of calorimeter material
$L$	characteristic length of calorimeter material, volume/area
$N$	number of observations
$p_i$	$i^{th}$ parameter in a mathematical model
$Q$	heat added to calorimeter per unit time
$S_{p_i}^T$	sensitivity coefficient, derivative of predicted temperature with respect to parameter $p_i$
$T$	temperature of lumped-parameter calorimeter
$T_i$	measured temperature at time $\tau_i$
$T_{i,cal}$	temperature calculated from mathematical model at time $\tau_i$
$T_s$	ambient temperature
$\Delta T_{max}$	maximum temperature rise above ambient
$t$	real time
$\alpha$	example time constant, in eq. (23)
$\beta$	dimensionless heating rate, $Q/hA \Delta T_{max}$
$\rho$	density of calorimeter material
$\tau$	dimensionless time, $ht/\rho LC_p$
$\tau_i$	discrete dimensionless time at which $T_i$ is measured
$\tau_m$	dimensionless duration of experiment
$\tau_o$	dimensionless time at which heater was switched off

$\tau_{\text{opt}}$  optimum duration of experiment

$\Phi$  dimensionless temperature of lumped-parameter calorimeter,  $(T - T_s)/\Delta T_{\text{max}}$

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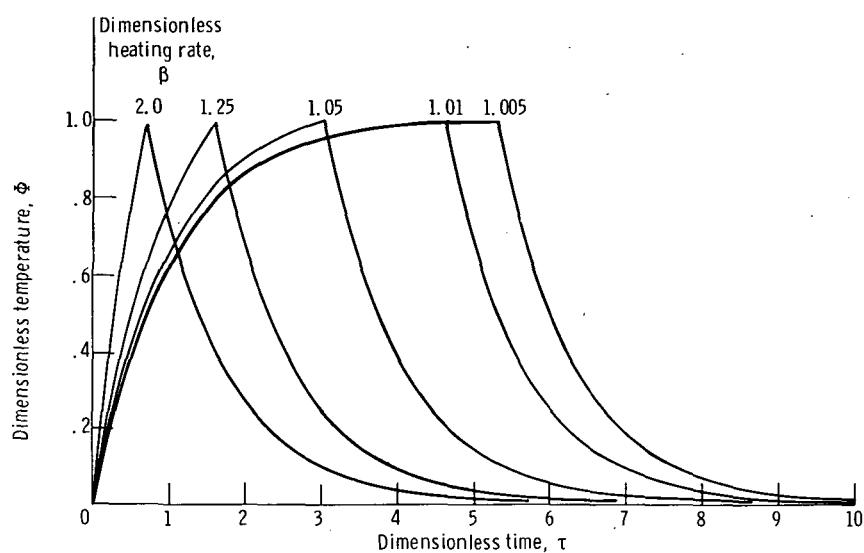


Figure 1. - Temperature as function of time for lumped-parameter calorimeter at several different heating rates.

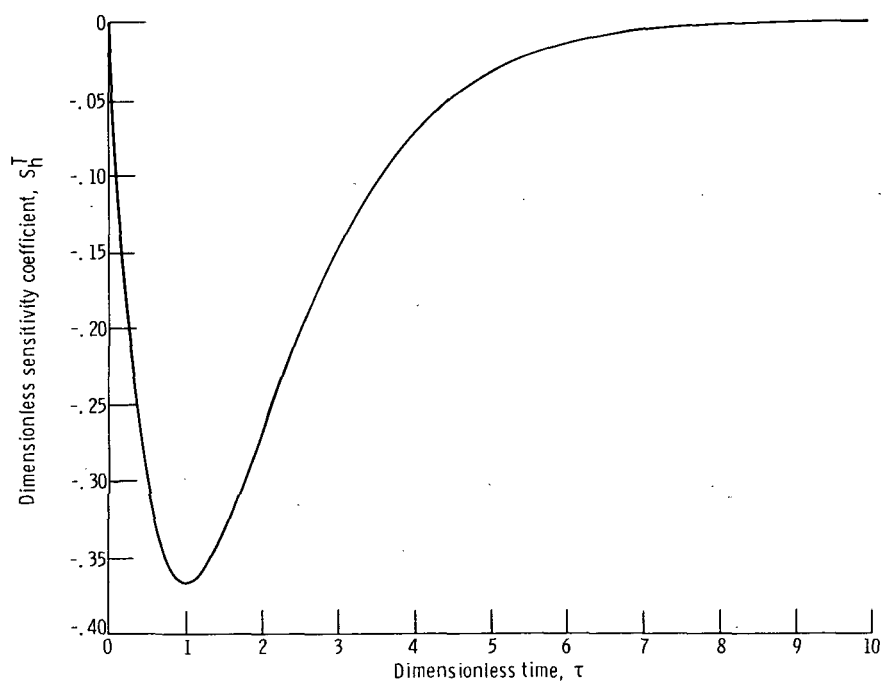


Figure 2. - Sensitivity coefficient as function of time for cooling only.



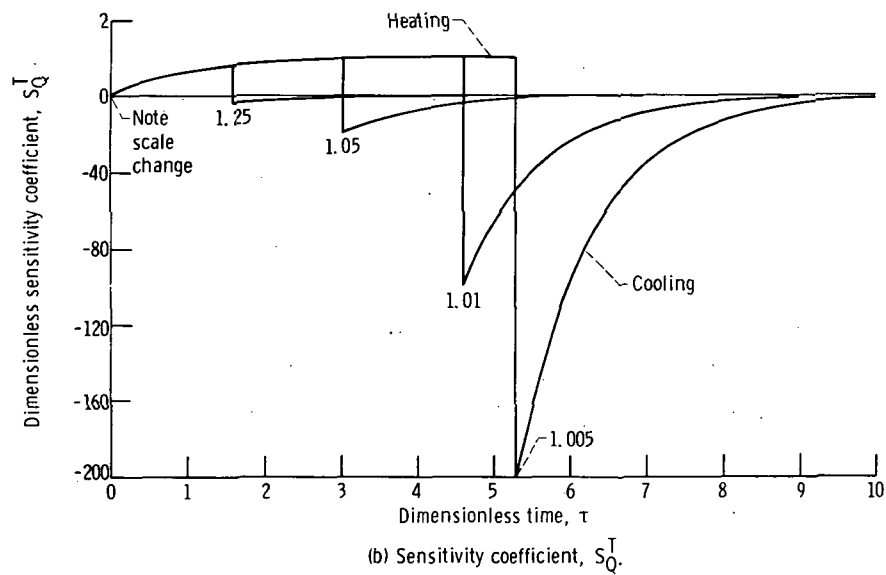
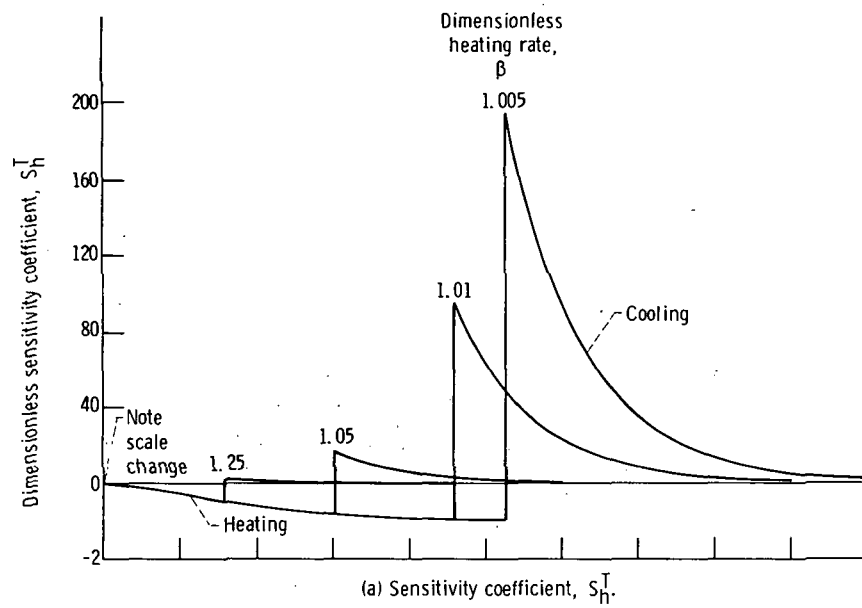


Figure 3. - Effect of heating rate on sensitivity coefficients for both heating and cooling.

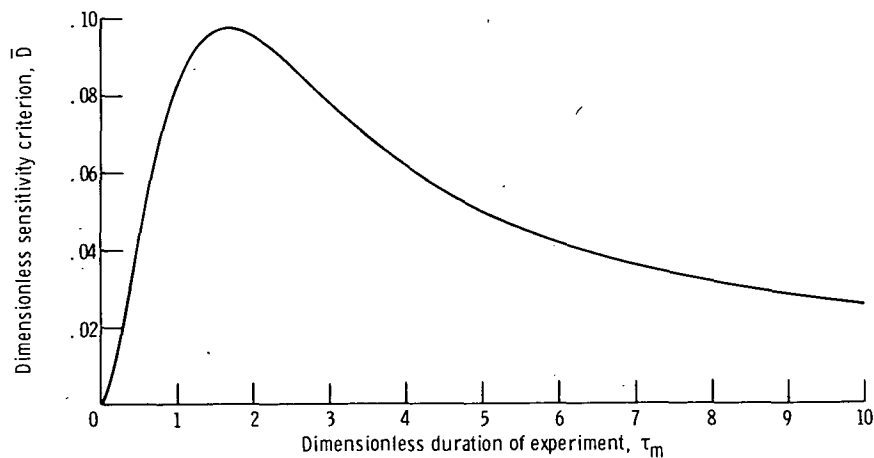


Figure 4. - Sensitivity criterion as function of duration of experiment for estimating heat-transfer coefficients from cooling data only.

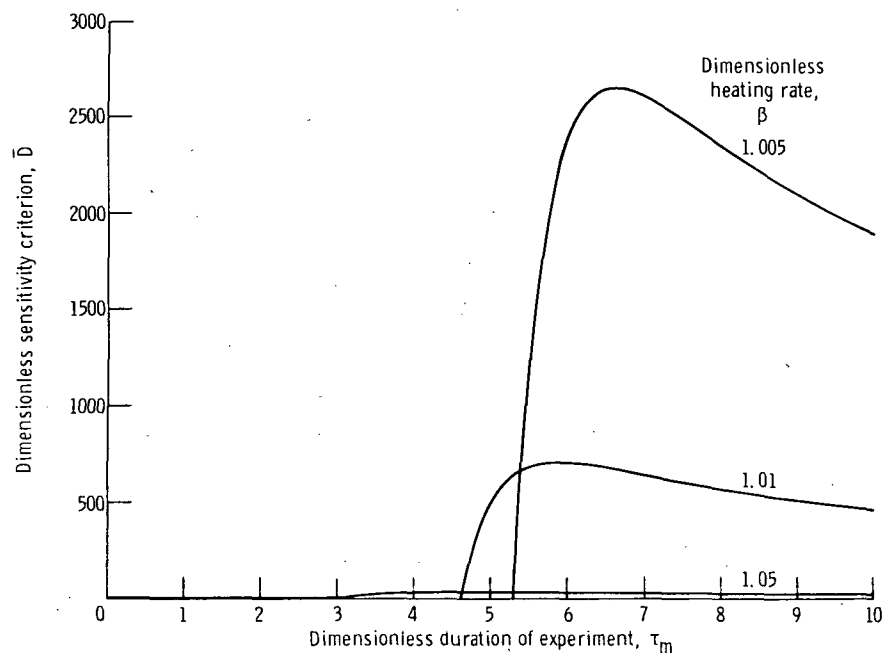


Figure 5. - Sensitivity criterion as function of duration of experiment for estimating heat-transfer coefficients from heating and cooling data.

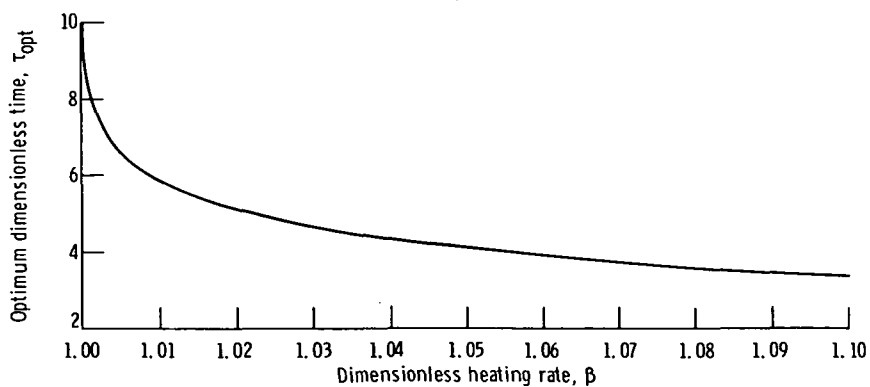


Figure 6. - Optimum dimensionless time as function of heating rate for estimating heat-transfer coefficients from heating and cooling data.

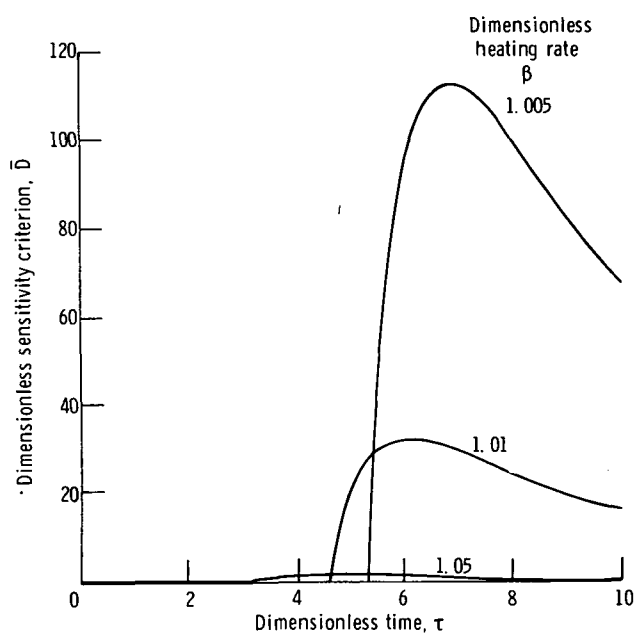


Figure 7. - Sensitivity criterion as function of time for estimating heat-transfer coefficients and heat addition from heating and cooling data at several heating rates.

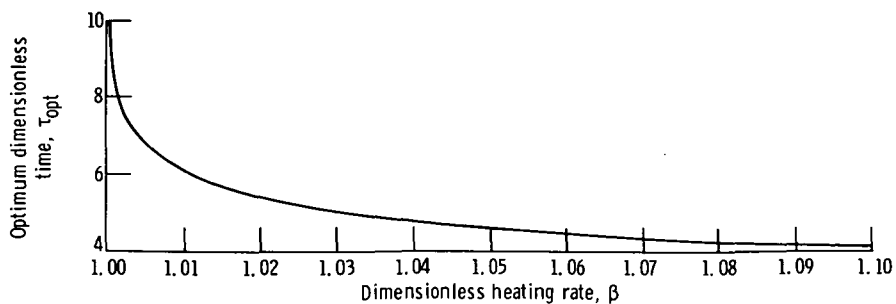


Figure 8. - Optimum time as function of heating rate for estimating both heat-transfer coefficients and heat addition from heating and cooling data.

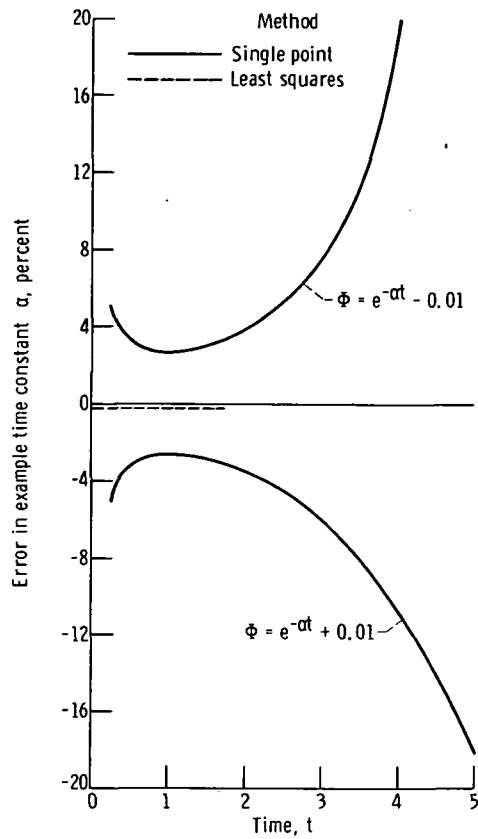


Figure 9. - Error in estimating example time constant with a 1 percent measurement error as function of time at which measurement is taken.

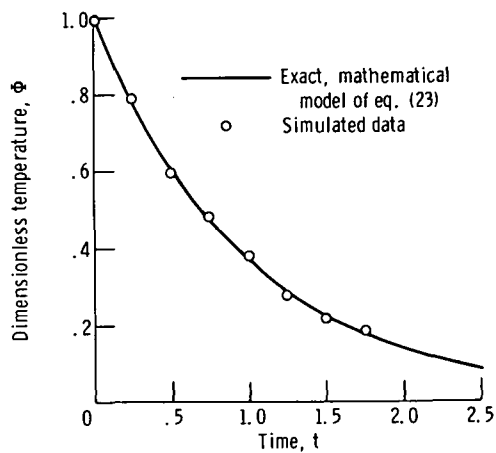


Figure 10. - Simulated data used in least-square example.

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